A Survey of Verifiable Delegation of Computations

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# Motivation
Cloud computing, Small Devices, Large Scale Computation

## Generic Results for Verifiable Computation
Protocols that work for arbitrary computations
- Interactive Proofs
- Probabilistically Checkable Proofs
- "Muggles" Proofs
- Other Arithmetizations approaches (QSP)
- Implementations (Pinocchio, Snark-for-C)

## Delegation of Memory
- Homomorphic MACs
- Proofs of Retrievability
- Verifiable Keyword Search
## Talk Outline

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Computing on Demand

Cloud Computing

- Businesses buy computing power from a service provider

Advantages

- No need to provision and maintain hardware
- Pay for what you need
- Easily and quickly scalable up or down

Trust Issues

- Transfer possibly confidential data to computing service provider
- Trust computation is performed correctly without errors
- Malicious or benign
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- Small devices outsourcing complex computing problems to larger servers
  - Photo manipulations
  - Cryptographic operations

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  - Confidentiality of data
  - Correctness of result
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Large Scale Computations

- Network-based computations
  - SETI@Home
  - Folding@Home

- Users donate idle cycles
  - Known problem: users return fake results without performing the computation
  - Increases their ranking

- Needed a way to efficiently weed out bad results
  - Currently use redundancy
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Verifiable Computation

- The client sends a function $F$ and an input $x$ to the server.

- The server returns $y = F(x)$ and a proof $\Pi$ that $y$ is correct. Verifying $\Pi$ should take less time than computing $F$. 

![Diagram showing a mobile device and a server exchanging inputs and outputs](image)
Verifiable Computation

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![Diagram showing the process of verifiable computation between a smartphone and a computer.](image-url)
Interactive Proofs (GMR,B)

- An all powerful Prover interacts with a poly-time Verifier
  - Prover convinces Verifier of a statement she cannot decide on her own
  - Probabilist guarantee
  - All of PSPACE can be proven this way [LFKN,S]

- We want something different
  - A scaled back version of this protocols for efficient computations
  - A powerful but still efficient prover: its complexity should be as close as possible to the original computation
  - A super-efficient Verifier: ideally linear time
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Muggles Proofs (GKR)

- Poly-time Prover interacts with a quasi-linear Verifier
  - Refines the proof that IP=PSPACE to efficient computations

- For a log-space uniform NC circuit of depth $d$
  - Prover runs in $\text{poly}(n)$
  - Verifier runs in $O(n + \text{poly}(d))$
  - Interactive ($O(d \cdot \log n)$ rounds)
  - Unconditional Soundness
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Optimizations and Implementations (CMT,T)

- Prover can be implemented in $O(S \log S)$
  - Where $S$ is the size of the circuit computing the function
  - $O(S)$ for circuits with a regular wiring pattern

- Implementation tests show that for the regular wiring pattern case the prover is less than 10x slower than simply computing the function.

- Protocol remains highly interactive
  - Interaction can be removed via the Fiat-Shamir heuristic (random oracle model).
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The Prover commits to this proof using a Merkle tree and then the Verifier queries it and verifies the openings (K)

Note that now we have an argument with a computational soundness guarantee

This protocol can also be made non-interactive using the random oracle (M) or strong extractability assumptions about the hash function used in the protocol (DL, BCCT, GLR)

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Arithmetization

- Turn a circuit computation into a set of polynomial equations
  - Replace each gate with a quadratic polynomial
  - Check these polynomial identities in a randomized fashion by checking them on random points
  - Use error-correcting encodings to make sure that the proof is *locally checkable* (i.e. to reduce the number of random queries to the proof)

- Can we use different arithmetizations?
  - Avoid composing long PCP proofs with compressing hash functions for a more direct way to get short proofs
  - Linear Prover complexity?

- Groth showed a different approach
  - Polynomial equations are verified in the exponent (using bilinear maps over a cyclic group)
  - A Diffie-Hellman type of assumption prevents the Prover from cheating
  - Proof is very compact without using Merkle trees
  - Drawback: quadratic prover complexity and a quadratic CRS
  - Lipmaa shows how to reduce those to quasilinear
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Quadratic Span Programs (GGPR)

QSPs add a single quadratic step to the computation, instead of checking several quadratic equations (one for each gate)

- To check that all the wires in the circuits are correct it just requires a linear test (span program)
- This would be too much work for the verifier (same as the size of the circuit)
- Build two copies of the "checking" span program and test them against each other
- A QSP is defined by two sets of polynomials $V = \{v_1, \ldots, v_{n+m}\}$, $W = \{w_1, \ldots, w_{n+m}\}$ and a target polynomial $t$
  - We say that a QSP $(V, W, t)$ computes a Boolean function $F$ of $n$ inputs if and only if
  - For all $x = (x_1, \ldots, x_n)$ s.t. $F(x) = 1$
    - $t$ divides the product of a linear combination of subsets of $V$ and $W$
    - $t | (\sum_{i=1}^{n+m} a_i v_i) \cdot (\sum_{i=1}^{n+m} b_i w_i)$
    - where $a_i = b_i = 0$ if $x_i = 0$
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The QSP protocol

- In a preprocessing stage the Verifier publishes the values $g^{s_i}$, $g^{v_i(s)}$, $g^{w_i(s)}$ and $g^{t(s)}$
  - for a secret random value $s$.
- On input $x$ the server finds the coefficients $a_i$, $b_i$ and polynomial $h$ such that
  - $t \cdot h = (\sum_{i=1}^{n} a_i v_i) \cdot (\sum_{i=1}^{n} b_i w_i)$
- Using the values produced by the Verifier the Prover can evaluate in the exponent the above equation at the point $s$
  - Verifier checks the equation using bilinear maps
- Efficiency:
  - The verifier is linear to prepare the input; constant time to verify the result
  - Prover is quasi-linear - the polylog overhead comes from doing polynomial division to compute $h$
- Security: requires a Diffie-Hellman type of assumption which assumes that the prover cannot divide in the exponent.
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Implementation Results

**Pinocchio (PGHR)**
- An end-to-end toolchain that compiles a subset of C into QSPs
- Proof size is 288 bytes regardless of what it is being computed
- Verification time is 10ms
- Prover complexity still not quite there in practice
  - About 60 times faster than previous proposals
  - Can run some lightweight computations

**SNARKs-for-C (BCGTV)**
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- Use the generic concept of Linear Interactive Proof
- Could plug a more efficient LIP if one is found
- Slightly less efficient for the Verifier
- Proof size: 322 bytes
- Verification time dependent on x (from 103ms to 5s for long inputs)
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- Were able to handle a Traveling Salesman Decider on a 200-nodes
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  - Then the TinyRam code is compiled into a circuit
- A QSP is built for this circuit
  - Use the generic concept of Linear Interactive Proof
  - could plug a more efficient LIP if one is found
- Slightly less efficient for the Verifier
  - Proof size 322 bytes
  - Verification time dependent on $x$ (from 103ms to 5s for long inputs)
- A bit more efficient for the Prover
  - Were able to handle a Traveling Salesman Decider on a 200-nodes graph
Outsourcing Your Data

- Up to now we have considered the case of a client sending $F$ and $x$ to the server
  - Client’s limitation is in computing time
  - Cannot compute $F$ on its own

- What if the client’s limitation is storage?
  - Client stores large quantity of data $D$ with the server
  - later queries $F$ on $D$ and receives back $F(D)$

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Homomorphic Message Authenticators (GW)

- Client stores $D = D_1, \ldots, D_n$ and $t_i = MAC_k(D_i)$.
  - Client only stores the short key $k$.

- Later the client submits $F$
  - Server returns $y = F(D)$ and $t$
  - Client accepts if and only if $t = MAC_k(y)$
  - Verification time may be as long as computing $F$ – focus on storage and bandwidth

- Original idea uses homomorphic encryption
  - Mostly of theoretical interest

- New ideas use "traditional" crypto (CF,GN)
  - Much more efficient
  - But only work for "shallow" circuits
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Proofs of Retrievability (JK)

- Client stores a large file $F$ with the server and wants to make sure that it can be retrieved without downloading the entire thing (e.g. auditing)
  - Client sends a short challenge $c$
  - Server responds with a short answer $a$
    - avoid reading the entire file to produce the answer

- A possible solution (A+,SW)
  - Encode the file $F$ using an error correcting code $F' = \text{Encode}(F)$
  - Store each block $F'_i$ with a linearly homomorphic MAC
    $$t_i = \text{MAC}_k(F'_i)$$
  - The client queries a small number ($\ell$) of the blocks $F_{i_1} \ldots F_{i_{\ell}}$ and also sends $\ell$ random coefficients $\lambda_1, \ldots, \lambda_{\ell}$
  - The server sends back $\phi = \sum_j \lambda_j F_{i_j}$ and $t = \sum_j \lambda_j t_j$
  - The client accepts if and only if $t = \text{MAC}_k(\phi)$

- The scheme is very efficient
  - Linearly homomorphic MACs can be built from basic universal hash functions
  - Minimal storage overhead due to the error-correction expansion
  - Query complexity is quadratic in the security parameter
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Verifiable Keyword Search (BGV)

- Client stores a large text file $F = w_1, \ldots, w_n$ with the server
  - Client sends a keyword $w$
  - Server responds with yes/no
  - how can we efficiently verify the answer?

- Encode the file as the polynomial $F(X) = \prod_i (X - w_i)$
  - Note that $F(w) = 0$ if and only if $w \in F$

- Problem reduces to efficiently verifying the computation of a large degree polynomial.
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Verifiable Computation of Polynomials (BGV)

- **Other applications besides Verifiable Keyword Search**
  - Client stores a high degree polynomial $F(X) = \sum a_iX^i$
    - Client sends a value $x$
    - Server responds $y = F(x)$
    - how can we efficiently verify the answer?
  - Store the MAC $t_i = ca_i + r_i$
    - $r_i$ are computed pseudorandomly, i.e. $r_i = PRF_k(i)$
    - Client only stores random secret keys $c, k$
    - Let $R(X)$ be the polynomial defined by the $r_i$
  - When the client queries the value $x$, the server returns
    - $y = \sum a_ix^i$ and $t = \sum t_ix^i$
  - The client checks that $t = cy + R(x)$
    - Note that this requires $O(d)$ work where $d$ is the degree of the poly
    - This can be reduced if we use closed-form efficient PRFs
    - Knowledge of the key $k$ allows the computation of $\sum r_ix^i$ in $o(d)$ time
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Merkle-trees allow to check individual memory locations which change over time, but not "global" verifications (proof of retrievability, verifiable keyword search).

Some progress on dynamic proofs of retrievability (CW, SSP).
Dynamic Storage

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Future Directions

- **Multiple clients**
  - Protect information from the other clients
  - Becomes secure multiparty computation with an added constraint
    - only one party has enough resources to compute the desired functionality
  - Leverage successes in SMC.

- **General VC**: Explore more realistic models of computation
  - e.g. RAM

- Explore more pragmatic approaches
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- Does the outsourcing of polynomials have larger applicability?
  - Alternatively, can we use the same idea of "closed form efficient" PRFs for other computations

- A more efficient general result for memory outsourcing/homomorphic MACs

- "Important" Computations, which would benefit from being outsourced:
  - Image processing
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